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THE PROPERTIES OF WATER-FILLED TUBES FOR IMPEDANCE AND SPEED OF--ETC(U)
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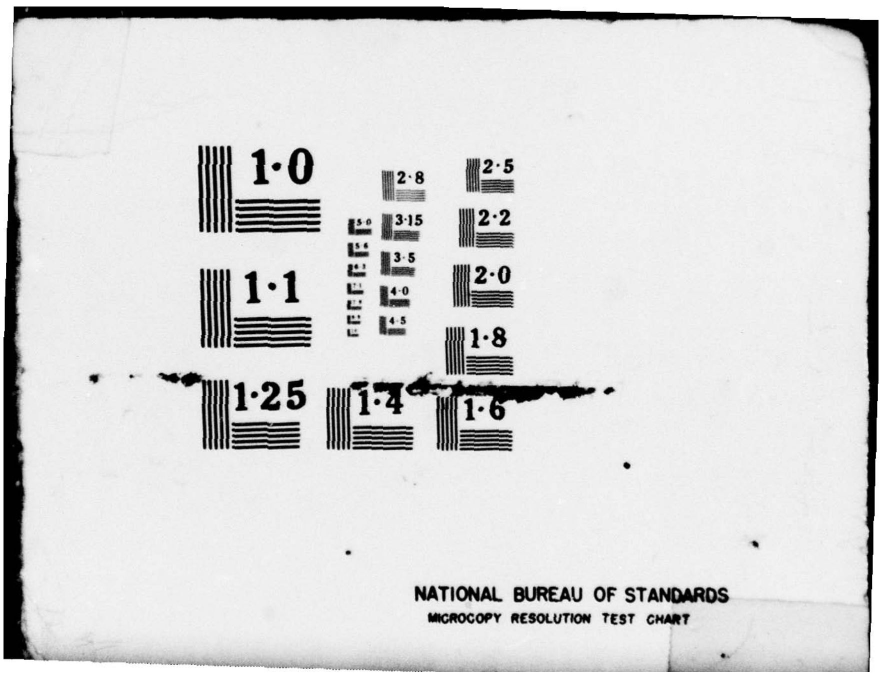
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THE PROPERTIES OF WATER-FILLED TUBES FOR IMPEDANCE AND SPEED OF SOUND MEASUREMENTS

by Walter Kuhl*

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Abstract: After a description of water-filled tubes for various acoustic measurements, the lowering of sound velocity for low frequencies by the resilient tube wall and the dispersion at higher frequencies is dealt with. Disturbances of sound propagation in the water are caused only by free vibrations of the tube, the excitation of which is diminished by filters for structure-borne sound. The natural frequencies of various kinds and orders of free tube vibrations not yet described in the literature are given.

As a significant number for the quality of the measuring tubes, the ratio of maxima and minima in tubes terminated without absorption was measured in a large frequency range. It amounts to 58 dB. In one case the sound insulation between the lower and the upper part of the tube, joined only by the tube wall not by the water column, was examined. Various kinds of rigid terminations of specimens under test and the required quality of microphones for exploring the sound field in the tubes are described.

The accuracy of the measurement of acoustic impedances is demonstrated by many examples (sound absorbers, microphones). The sound velocity was measured at various frequencies and air contents and an absolute accuracy of 0.7 to 1.5 x 10^{-3} was assured.

(Author's English summary)

1. INTRODUCTION

The measurement of acoustical impedance in tubes by means of standing waves has been well known for a long time and the method is frequently used. Such measurements in a water-filled tube were first reported by K. Tamm [1]. He used a tube whose wall was designed as a low-pass acoustical filter so as to prevent the propagation of structure-borne sound in the tube and thus its coupling to the water column. The tube had the disadvantage that tube resonances restricted the frequencies at which the

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filter action was effective to a very small range. It was therefore desired to learn whether, by greatly increasing the wall stiffness and using a tube material having a large modulus of elasticity, the coupling between the water and the tube could be so decreased that it would not disturb the sound propagation in the water, and accurate measurements would be possible in a wide frequency range.[1]

It was in tubes of this kind that impedance measurements were made on ribbed absorbing material [3] and resonant absorbers for use in water [4], and the frequency curves of microphones were measured. The sound speed in a tube was measured for various temperatures and various air contents, and good agreement with calculated values obtained. Tubes having circular and square cross sections lined with thin acoustically soft or sound-absorbing material could be used to measure the sound absorption in progressive waves, which will be reported elsewhere [5].

A series of measurements of sound reflection by absorbers, especially resonant absorbing material, can be made quicker, more accurately, and for various static pressures in long tubes the upper end of which can be closed, by using pulsed progressive waves instead of standing waves. A separate report on this subject will soon be published [6].

For the impedance measurements, the sound pressure was measured by a stiff crystal microphone at one or more minima and maxima, and the distance of the first minimum from the measurement sample was measured; from these values the complex impedance was determined in the familiar manner either mathematically or by means of a diagram, such as that by Tamm [1].

To obtain a large measurement accuracy, the waves must be propagated in the tube without distortion, which means without damping and without spurious signals such as might be radiated into the tube by the tube wall. For purely imaginary terminal impedance, that is for lossless mass or elastic impedances, the minima are zero because of the complete reflection of the sound pressure. These spots are therefore especially sensitive to disturbances. After describing the measurement system and discussing sound propagation in the tube, we will show where the disturbances to the sound propagation come from, how they can be kept small, and how great the attainable measurement accuracy is.

2. MEASUREMENT SYSTEM

The acoustical part of the measurement system consisted of a water-filled tube with the upper end open, on which, generally at the bottom, a magnetostrictive or Rochelle salt sound source was fastened, either with or without insertion of a filter for structure-borne sound. Steel tubes were preferably cadmium-plated for rustproofing. A tourmaline microphone 6 mm in diameter and 6 mm long was used. The leads were attached through a brass tube of 2 mm diameter that was screwed into a preamplifier; the entire assembly could be arbitrarily adjusted in height. The characteristics

[1]Between the completion of this article and its publication, an article by R. D. Fay, R. L. Brown, and O. V. Fortier [2] has been published in which similar investigations and calculations are reported.

of such a microphone are more fully discussed later. When the measurement samples are placed at the upper end of the tube, the microphone leads must be introduced through a hole (4 mm diameter) bored in the sample. To keep the variations of the water height small when the sample disks are changed, and to be able to neglect them completely for raising or lowering the microphone, the lower end of the tube was connected by a rubber hose to a compensating reservoir of very large cross section. The measurement tube was enclosed in a concentric cylinder. A temperature-controlled fluid could flow through the space between.

The free upper surface of the water served as an acoustically soft termination for the calibration. A sample immersed in the water up to its upper surface also has an acoustically soft termination (short circuit).

To provide an acoustically hard termination for a sample (open circuit), the upper surface of the sample is spaced a quarter wavelength from the free upper surface of the water or the tube is terminated at the lower end with a large mass, for which it is not necessary to bore a hole through the sample. The necessary cushioning of the mass against the tube is more thoroughly discussed later. When the tube is terminated in this way at the lower end, a small sound source is immersed in the water at the top or fastened to the side of the tube.

3. SOUND PROPAGATION IN THE TUBE

a) Waveforms and Limiting Frequencies

The wall of the measurement tube should be as rigid as possible, so that calculation of the sound propagation for a rigid tube will be a good approximation. In tubes with the inner diameter $2r_1$, only a plane wave is propagated below a lower frequency limit $f_{gr} = 0.585c/2r_1$. In a tube with a square cross section and inside width a, the corresponding frequency is $f_{gr} = c/2a$. Above the lowest limiting frequency, several frequencies can be found for airborne sound up to the next following limiting frequency for which the disturbance is slight through the first of the higher wave types. For waterborne sound, because of the strong sympathetic vibrations of the tube wall, complete symmetry of excitation is very much more difficult to achieve and therefore a measurement is hardly possible with standing waves above the lowest limiting frequency. If pulsed progressive waves [6] are used, a separation in time of the disturbing pulses from the desired pulses of the plane wave is possible under certain conditions because of the reduction of the group velocity of the higher wave types, and therefore a reflection measurement is possible.

b) Reduction of the Sound Speed for Low Frequencies because of the Compliance of the Tube Wall

For airborne sound, tubes made of solid materials can be assumed to be very rigid, if the resonance frequencies of thin-walled tubes are excluded. Sound propagation within the tube is thus almost independent of the tube itself. This is not the case, however, for the slight difference between the wave impedances of water and solids. One must consider the effect of

the tube wall, which manifests itself first in a reduction of the sound speed in the tube compared to that in the free medium.

Korteweg [7] was the first to calculate the speed of sound in a fluid-filled circular cylinder with compliant walls. Because of the compliance of the tube, the cross section of the tube is increased by a pressure inside. Therefore the decrease of the thickness of a thin water layer in a plane perpendicular to the tube axis is larger by an amount that corresponds to the compressibility of the unbounded liquid. The increase of the compressibility means a lowering of the sound speed. Let

r_1 = inner radius of the tube

r_2 = outer radius of the tube

$h = r_2 - r_1$ = wall thickness of the tube

E_w = modulus of elasticity of the tube wall

$E_0 = \rho c^2$ = modulus of volume elasticity of the fluid

ρ = density of the fluid

c = sound speed in the unbounded fluid

c' = sound speed in the fluid in the tube.

The tangential stress on the inner rim of a very thin-walled tube, for internal pressure p, is

$$\sigma_{t_1} = pr_1/h = \frac{\Delta r_1 E_w}{r_1} . \qquad (1)$$

Proceeding from this hypothesis, Korteweg gives the sound speed in a fluid enclosed in a very thin walled tube as

$$\frac{c'}{c} = \left[1 + \frac{2E_0}{E_w} \cdot \frac{r_1}{h}\right]^{-\frac{1}{2}} . \qquad (2)$$

For greater wall thickness, the tangential stress in the tube wall decreases from the inner to the outer edge. The outer layer of the wall contributes less than the inner to the stiffness of the wall, so that the effective wall thickness must be assumed smaller than the actual thickness.

After introducing a correction[2], Korteweg obtained the following formula for the sound speed:

$$\frac{c'}{c} = \left[1 + \frac{2E_0}{E_w} \; \frac{r_1}{h(1 - 5h/6r_1)}\right]^{-\frac{1}{2}} . \qquad (3)$$

[2] The correction does not correspond to a difference between the static and the dynamic modulus of elasticity of the tube wall material as Field [8] erroneously states.

Ganitta [9] confirmed the formula by measurements in thin-walled water-filled tubes of glass and metal ($h/r_1 \leq 0.2$).

In a tube with arbitrary wall thickness, the tangential stress at the inner rim is [10]

$$\sigma_{t_1} = p \, \frac{(r_2/r_1)^2 + 1}{(r_2/r_1)^2 - 1} \, . \tag{4}$$

If Eq. (4) is used instead of Eq. (1), a more accurate formula for the relative sound speed is obtained:

$$\frac{c'}{c} = \left[1 + \frac{2E_0}{E_w} \cdot \frac{(r_2/r_1)^2 + 1}{(r_2/r_1)^2 - 1} \right]^{-\frac{1}{2}} \tag{5}$$

Thus, the reduction of the sound speed in the tube is a function of the ratio of the modulus of elasticity of the wall material to the modulus of volume elasticity of the fluid and the ratio of wall thickness to inner radius. Fig. 1 is a plot of c'/c from Eq. (5) for steel as the tube material ($E_w = 2.00 \times 10^6$ kg/cm^2) as a function of the relative wall thickness h/r_1. Also, for infinitely large wall thickness ($h/r_1 \rightarrow \infty$), c' is smaller than c because of the compressibility of the wall material. The limiting value for steel is $c'/c = 0.9892$; for brass, it is 0.983.

In the Korteweg statement of the formula as well as the improved form (5) for arbitrary wall thickness, substantial approximations are made in that the longitudinal stress in the tube wall and the flexural stiffness of the tube wall are not considered and it is assumed that each ring element of the tube wall oscillates according to the locally existing sound pressure independently of the neighboring element, which certainly is not the case. Nevertheless, for very large wall thicknesses the agreement with measurements quoted later (see Table II) is surprisingly good.

Other theoretical articles by Lamb [11], Bugentzianu [12][13], and Gronwall [14], concerning the reduction of the sound speed in the fluid because of the compressibility of the tube wall present almost the same curve of c'/c as a function of h/r_1 as that obtained from Eq. (5). As a consequence of considering the transverse contraction for finite tube length, the limiting value of c'/c for $h/r_1 \rightarrow \infty$ in a steel tube is reduced to 0.9864. The values calculated from the most complete theory--that of Gronwall--are also shown in Fig. 1. For $h/r_1 = 0.1$, the values by Lamb agree with the curve from Eq. (5), those by Bungentzianu lie 0.6% above it.

The compliance of the tube wall is doubly disadvantageous for the measurement of samples, which makes a large wall thickness urgently necessary. In tubes with slight wall thickness, the compressibility of a

measurement sample is somewhat changed by the compressibility of the tube wall. Most important, however, the compressibility of the tube wall reduces the speed of sound in the gap between a measurement specimen in the tube and the tube wall by a substantially larger amount than it does the sound speed in the tube, since the deflection of the tube wall (proportional to the sound pressure) is constant, but the volume of the gap for equal length is substantially smaller than that of the tube. The greatly reduced sound speed in the gap has undesired consequences, which will be treated more precisely by Kuhl, Oberst, and Skudrzyk [6].

c) Dispersion of the Sound Speed

When the wavelength in water at high frequencies is no longer large compared to the internal tube diameter, then the sound pressure is no longer constant over the whole cross section as the simple theory assumes. An accurate plot of the sound pressure and particle velocity distribution requires consideration of the effect that Kuhl and Tamm [5] discuss in more detail, as well as the frequency-dependent tube impedance. If only the elasticity of the tube wall is considered, one obtains with increasing frequency a slight decrease of the speed of the plane wave (Figs. 3 and 4; see also [32]) that slowly deteriorates since an increasing radial component is added to the longitudinal component of the particle velocity. Wave propagation remains possible, however, and the sound speed retains a finite value.

When the mass impedance of a ring-shaped element, which increases proportional to the frequency, is likewise considered, a substantially larger fall-off of the compliance with the frequency is obtained. The two components of the impedance compensate each other for the radial longitudinal vibration of the ring. If the ring-shaped wall elements of a tube can vibrate independently of each other, the tube wall is acoustically soft for the frequency of the radial longitudinal vibration of the individual rings. The speed of the plane wave is then zero, and for higher frequencies it is imaginary. This assumption is only approximately satisfied for a tube like that shown in Fig. 2 whose wall is subdivided by machined grooves.

Field [8] applies a resonance impedance for the calculation of the sound speed in liquid-filled tubes as well as for the tube wall that is not subdivided. Our own measurements (Figs. 4, 5, 6) and those of others have shown that this is not permissible, since the sound speed does not go to zero.

The compliance of a tube like that shown in Fig. 2 arises from two sources. One is the increase of the tube circumference because of an internal pressure (first term in the square brackets in Eq. (6)). The corresponding elastic-impedance for the radial longitudinal vibrations is zero, if the mass of the tube wall is brought into consideration. The second source is the buckling of the thin tube wall portion (second term in the square brackets). The frequency of its lowest flexural resonance lies essentially higher than that of the radial resonance of the rings, so that only the compliance must be reckoned with. For low frequencies, the following formula holds for the sound speed in the tube

$$\frac{c'}{c} = \left\{1 + \frac{2E_0}{E_w}\left[\frac{(r_2/r_1)^2 + 1}{(r_2/r_1)^2 - 1}\left(1 + \frac{a}{b}\right) + \frac{a^5}{60r_1(a + b)h_1}\right]\right\}^{-\frac{1}{2}} \quad (6)$$

With the dimensions given in Fig. 2 and $E = 0.90 \times 10^{12}$ dyn/cm^2, the ratio $c'/c = 0.852$ is obtained. This value was accurately measured for low frequencies.

The frequency-dependent wall impedance was also taken into consideration in calculating the sound speed. The value of the curve b in Fig. 3 was obtained from Eqs. (7), (8), and (9) (see also [5]):

$$k' = \omega/c' = (k^2 - \gamma^2)^{\frac{1}{2}} = [(\omega/c)^2 - \gamma^2]^{\frac{1}{2}} \quad (7)$$

$$(\gamma/i)r_1 = \varkappa \quad (8)$$

$$i\varkappa\frac{J_1(i\varkappa)}{J_0(i\varkappa)} = \frac{\omega^2 r_1 \rho}{E_w}\left[\frac{(r_2/r_1)^2 + 1}{(r_2/r_1)^2 - 1}\cdot\frac{(1 + a/b)}{(1 - f^2/f_d^2)} + \frac{a^5}{60r_1(a + b)h_1^3}\right]\cdot \quad (9)$$

In Eq. (9), as already mentioned above, the mass of the tube wall is taken into consideration through the resonance factor $(1 - f^2/f_d^2)$ (f_d = resonance frequency of the radial longitudinal vibration of a ring[3]) for the first of the two parallel-connected compliances, and the second is assumed to be a pure spring. For calculating curve a, the resonance factor in Eq. (9) was neglected. The very good agreement with the measured points plotted in Fig. 3 shows that a subdivided tube is acoustically soft for the radial longitudinal vibration of the individual rings and the sound speed of the plane wave goes to zero. Above the frequency 29.5 kc, because of the occurrence of the first order of the higher wave types, reliable measurements can no longer be made.

For tubes with homogeneous wall, the ratios are different. Such tubes of finite length cannot execute radial longitudinal vibrations with constant amplitude and phase over the entire tube length; on the contrary, the radial longitudinal vibration of a short ring is transformed with increasing length into the longitudinal fundamental vibration of the tube. Then the above-described dispersion of the sound speed in the water in the tube drops out.

The curves in Fig. 4 apply to a glass tube (r_1 = 9.7 mm; h = 1.3 mm; $E_w = 6.03 \times 10^{11}$ dyn/cm) filled with petroleum (c = 1260 m/sec; p = 0.74). Curves b, which were calculated under the assumption that the tube wall vibrates in resonance at the frequency f_d of the natural radial frequency

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[3]The natural frequency for rings with slight wall thickness is $f_d = 1/\pi(r_1 + r_2)(E_w/\rho_w)^{\frac{1}{2}}$. For arbitrary wall thickness, it can be calculated by the exact theory of Žaček and Petržílka [15], the validity of which was experimentally demonstrated by Kuhl [16].

of a ring of the same dimensions, are taken from Field and Boyle [17], as were also the not very accurate measured values. Curves a were calculated from Eqs. (7) and (10) or (8) and (10a), in which only the compliance of the tube was included.

$$\gamma r_1 \frac{J_1(\gamma r_1)}{J_0(\gamma r_2)} = \frac{\omega^2 r_1 \rho}{E_w} \cdot \frac{(r_2/r_1)^2 + 1}{(r_2/r_1)^2 - 1}, \qquad (10)$$

$$i\varkappa \frac{J_1(i\varkappa)}{J_0(i\varkappa)} = \frac{-\omega^2 r_1 \rho}{E_w} \cdot \frac{(r_2/r_1)^2 + 1}{(r_2/r_1)^2 - 1}. \qquad (10a)$$

Since curve a for the plane wave agrees substantially better than does curve b with the measured points, which is also confirmed by the second measurement example by Field and Boyle, it is demonstrated that the tube wall can be considered as a pure spring and the mass impedance need not be taken into account. The same must hold for the higher wave orders. The measured points for this, however, agree better with the curve b from the Field and Boyle data. Boyle, Froman, and Field [18][19] have measured exclusively the plane wave up to high frequencies. Its speed increases with increasing frequency from the value $c' < c$ to c, the sound speed in the free medium. This increase cannot be explained by any of the existing theories.[4]

In the measurement of the characteristic sound speed in steel tubes with rather large wall thicknesses (Figs. 5 and 6; more details in Table II), the increase from c' to c, or at least its beginning, was determined. Measurements with usual accuracy could not be made above the lowest transverse vibration.

d) The Effect of Sound Absorption and Structure-borne Sound on the Development of Standing Waves

In airborne sound, one must consider the sound absorption in the tube that arises from the damping in the medium at high frequencies and wall friction at low frequencies. Neither form of attenuation plays any role

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[4]There exists no quantitative theory for the propagation of longitudinal vibrations in thick-walled tubes. It is to be assumed that they behave like solid cylinders [20][21]. For them, the speed of longitudinal waves decreases with increasing frequency from $c = (E/\rho)^{\frac{1}{2}}$ to about $0.6(E/\rho)^{\frac{1}{2}}$, and indeed in the same frequency range for which the speed of sound increases in a liquid enclosed in a thick-walled tube of a given diameter. The mode of vibration of the cylinder in this range is so complicated and varies so much with the ratio of the diameter to the wavelength that constant amplitude and phase of the radial displacements cannot be used to calculate the effect of the tube wall vibrations on the sound propagation in the water. If the wavelength is smaller than the diameter, then the displacements are confined more to the outermost wall element (Rayleigh wave). This is perhaps the explanation for the increases of sound speed in thick-walled tubes with increasing frequency that permits the conclusion that the compliance of the tube wall is small.

in waterborne sound. At the frequency 10 kc, a sound pressure loss of 1 dB first begins to occur in water free of air bubbles at distances of 50 to 100 km. The damping because of the friction of the medium on the tube wall can be computed from the Kirchhoff formula:

$$\alpha = 8.68(1/r_1 c)[\pi\mu/\rho]^{\frac{1}{2}}(f)^{\frac{1}{2}}. \quad (11)$$

For $r_1 = 2.5$ cm and $f = 10$ kc, the attenuation is 1 dB at about 22 m. That means, for example, that for a measurement length of 30 cm, the smallest difference in level between maxima and minima will be 62.5 dB.

Also, sound radiation into the air at the open end of the tube can be ignored ($20 \log p_{max}/p_{min} = 70$ dB) because of the large difference of the wave impedances.

If one puts into Eq. (5), as Ganitta [9] does, a complex modulus of elasticity $\bar{E} = E(1 + i\eta)$ for the wall material in order to take its loss into account, then the absorption of the sound in the liquid because of the coupled vibrations of the tube wall is given by

$$\alpha = 2\pi c' \frac{\rho}{E_w} \frac{(r_2/r_1)^2 + 1}{(r_2/r_1)^2 - 1} \eta f. \quad (12)$$

The attenuation is proportional to the loss factor η and the frequency. It becomes greater, the smaller the ratio of the outer to the inner radius becomes. For a steel tube with $h/r_1 = 0.6$, $c = 1450$ m/sec, $\eta = 1.0 \times 10^{-4}$ (as in our experiments) and $f = 10$ kc, the attenuation is $\alpha = 0.9$ dB/km. For rigid tubes, therefore, this attenuation also does not play any role. The acoustical disturbances to measurements with standing waves in a tube that must be considered, then, are sound propagation from the tube wall into the water column, especially from natural resonances of the tube, and sound conduction in the microphone lead stem, which will be discussed later.

If a disturbing signal of sound pressure p_{st} is introduced by the tube wall at the location of a pressure node of the standing wave, then the disturbance to the standing wave depends on the phase angle between p_{st} and the sound pressure in the standing wave. For equal or opposite phase, the pressure node is displaced $\sigma_{st}\lambda/4$ from the undisturbed condition. For $\pm 90°$ phase shift, the minimum is not displaced, but the sound pressure is not less than p_{st}. For other phase angles, a smaller displacement occurs together with a finite minimum sound pressure $p_{min} < p_{st}$. Table 1 shows several examples for various magnitudes of the ratio of p_{st} to the sound pressure p_e of the incident wave. If, in determining the sound pressure distribution in a tube having a loss-free termination, one finds a minimum difference between p_{max} and p_{min} of 32 dB, for example, that means a maximal error in the measurement of amplitudes of 5% of the sound pressure of the incident wave and a maximal error for σ of 0.016.

4. NATURAL RESONANCES OF TUBES AND MEANS OF PREVENTING THEIR EXCITATION

Since the acoustical excellence of the tube is based first of all on the excitation of the tube wall into resonance, let us briefly discuss the principal modes of vibration and the associated frequencies. The chief types of tube vibrations are the longitudinal vibrations, the axi-symmetrical flexural vibrations, and the two- and three-dimensional flexural vibrations.

The resonance frequencies of the longitudinal vibrations satisfy the relation $f_{1m} = c/2ml$, from which we obtain $m \geq 1$ for the number of nodes in the total length l. The axi-symmetrical flexural waves have constant vibration phase and amplitude on the circumference, but the direction of vibration rotates periodically along the cylinder. The number of nodes in the tube length is $m \geq 2$. A circular ring can execute two-dimensional flexural vibrations in its plane with 2n nodes on the circumference ($n \geq 2$). (See [16] for theories and numerous measurements of this.) It is advisable to choose the ratio h/r_1 in the range 0.8 to 3 for a given inner radius so as to put the frequency of the lowest flexural vibration as high as possible. It is in every case, however, smaller than the frequency of the lowest cross-sectional resonance in the tube. In addition to these two-dimensional ($m = 0$) vibrations, three-dimensional flexural vibrations can also occur with 2n nodes on the circumference ($n \geq 2$) and $m \geq 1$ nodal circles in the entire length whose characteristic frequencies are higher than those of the corresponding two-dimensional ones. The longer the tube length, the more closely spaced they are, especially for small orders m. For long tubes, the difference between the resonance frequencies for the two lowest modes ($m = 0$ and $m = 1$) is often so small that they cannot be separated from each other.

The characteristic frequencies of the flexural vibrations with 4 and 6 nodes on the circumference for 10 or 14 different orders m are shown in Fig. 7 for one of the tubes studied that had circular cross section (Tube No. 3) and whose dimensions can be taken from Table II. The lowest 13 of these characteristic frequencies fall within the acoustically useful frequency range below the lowest cross-section resonances in water.

Tubes with square inner and outer cross-section boundaries have three types of flexural vibrations with $n = 2$ and $m \geq 0$. Tube No. 2 (see Table II) with square inner cross section and circular outer boundary of the cross section exhibits two types of flexural vibrations with $n = 2$ and $m \geq 0$. Their mode of vibration is shown by the sketch in Fig. 8, in which the characteristic frequencies are plotted against the number m of the nodal circle. The lowest characteristic frequencies decrease by about 1% when the tube is filled with water. Of the characteristic frequencies shown in Fig. 8, 20 lie in the acoustical measurement range of the tube.

In order to keep the number of resonance frequencies of a measurement tube as small as possible, the tube should be short and thick-walled. The minimum length is determined by the lowest required measurement frequency, and it must be at least 3/4 of the wavelength in water.

A magnetostriction oscillator with a resonance frequency of 21 kc was generally used in the tube for measurements with standing waves. Rubber

rings were interposed as shown in Fig. 9 for attaching the source to the tube.

The vibrations of the tube walls were measured for various tubes that did not contain water by the use of a crystal sound pickup. The rubber rings reduced the amplitudes of all the resonance frequencies by about 35 to 50 dB compared to the direct attachment of the source when the resonances lie above the limiting frequency of the filter for structure-borne sound. The attenuation of the structure-borne sound is only 25 dB for the lowest longitudinal vibrations. To keep these vibrations also away from the tube, an additional filter for structure-borne sound with a characteristic frequency below 1 kc consisting of two rings fastened together with a brazed bronze sheet as shown in Fig. 10 could be inserted. In this way, the excitation of all tube vibrations by structure-borne sound was prevented.

All natural vibrations, however, including the non-axi-symmetrical flexural vibrations, are excited by the sound pressure in the water when the tube is water filled. The resonance amplitudes of the tube wall, because of the coupling through the water, are greater than without water filling; nevertheless, they are about 18-25 dB less than they would be if the source were directly bolted to the tube without the insertion of any filter for structure-borne sound. They fall about 20-50 dB below the amplitude of the source. By selecting a larger wall thickness, the vibration amplitudes can be reduced, but their excitation cannot be completely prevented. Embedding steel tubes in a material with higher attenuation (for example, lead with $\eta = 2.5 \times 10^{-3}$) has not been successful in our experiments.

An attempt was also made to prevent conduction of the structure-borne sound through the tube wall by dividing the wall in a lengthwise direction so that it would act as a mechanical low-pass filter. For this, the tube wall can be provided with grooves [1] as shown in Fig. 2, so that the sections of large wall thickness act as masses, the sections between act as springs, with the mass and spring combination determining the cut-off frequency above which the blocking occurs. The blocking is of course effective only for the transmission of pure axial displacements, thus only for one of the several degrees of freedom of the individual elements. Because of this, the blocking action is confined to the frequency range between the cut-off frequency and the nearest following natural frequency of the individual rings, which cannot be made arbitrarily large. Since thick-walled steel tubes are quite suitable in many respects, tubes with a subdivided wall were abandoned.

5. THE ACOUSTICAL PROPERTIES OF SOME MEASUREMENT TUBES

To determine the acoustical properties of three measurements tubes, about which more complete information is given in Table II, the speed of sound was measured in all the tubes over a larger frequency range. Moreover, the ratio of the maxima of the sound pressure of the standing wave to the minima were measured in tubes Nos. 1 and 3, and the sound attenuation between the lower and the upper parts of tube No. 2 were measured for sound transmission through the tube wall only.

The accuracy of the measurement of the sound speed was $\pm 2 \times 10^{-3}$ for these measurements. It was practically equal to the reading accuracy of the AEG frequency meter which was used for the frequency measurement. The frequency meter was calibrated against a standard frequency of 10 kc $\pm 10^{-8}$ as well as multiples and fractions thereof. The accuracy of the wavelength measurement was $\pm 5 \times 10^{-4}$.

The sound speeds measured at the low frequencies (Table II) agree within $\pm 2 \times 10^{-3}$, that means they agree with the calculated values within the limits in which the speed in the free medium and the reduction of the sound speed by the material properties of the tube are known. The sound speeds in the steel tubes Nos. 1 and 2 are plotted as a function of frequency in Figs. 5 and 6, previously mentioned.

A suitable figure for characterizing the quality of measurement tubes with respect to their usefulness for measurements with standing waves is the ratio of p_{max} and p_{min} in the water column with the upper end open, thus with a lossless termination. This ratio is plotted in Fig. 11 for the tube No. 1, turned entirely from steel with $h/r_1 = 0.6$, over the complete frequency range that is useful to the measurements. The solid-line curves were obtained without use of the second filter for structure-borne sound shown in Fig. 10. The curve obtained after inserting this filter coincides almost exactly with the solid-line curve up to the dotted portion. For the lowest longitudinal natural vibration of the tube wall (f = 6.35 kc), the ratio p_{max}/p_{min} was reduced by 8-13 dB without the added filter for structure-borne sound. The decoupling of the tube from the source by means of the filter raised the natural frequency to 6.73 kc. The difference between p_{max} and p_{min} then amounted to 21-35 dB.

Only in a few narrow frequency ranges, which altogether make up only a fourth of the entire range, was the difference in level of p_{max} and p_{min} smaller than 2% of the sound pressure p_e of the incident wave for the amplitude measurement, and smaller than $0.006\lambda/4$ for the measurement of the displacement of the minimum. At the worst spot, the errors amounted to $0.1p_e$ and $0.03\lambda/4$. The tube could have been improved still more by increasing the wall thickness.

The hollow brass tube was substantially less satisfactory because of the larger compliance of the tube wall and the greater number of resonances, as Fig. 12 shows. The amplitude error for measurements with standing waves amounted to about 5 to 25%.

Another method was chosen for the investigation of tube No. 2 which was made entirely of steel and had a square inner cross section and circular outer boundary. Because this tube served mainly for the measurement of the attenuation of sound propagation when one or two walls were covered with acoustically soft or sound-absorbing material and of the attenuation of filters [5], the acoustical "shunt" through the tube wall was investigated. For this purpose, a short section of the tube half-way along its length was filled with cellular rubber so that the two parts of the tube were almost completely separated acoustically from each other. Absorbing wedges were

applied below the cellular rubber and at the upper end of the tube so that a progressive wave could be produced in both parts. The ratio of the sound pressures of the two waves is equal to the sound absorption between the lower and the upper parts of the tube, which are coupled only through the tube wall. Figure 13 shows the result of this measurement. The measurable resonances for the water-filled tube are also recorded here. Their number is larger than for tube No. 1 because of the larger tube length (500 instead of 400 mm) and because of the increased degrees of freedom, although the average wall thickness is smaller. Between 10 and 20 kc, 35 resonances were found for the empty tube. They were undesirable. The extreme values of the sound absorption were 20 and 57 dB. The average value decreased from 50 dB at 2 kc to 30 dB at 20 kc. The values for p_{max}/p_{min} would lie 6 dB above the curve. A comparison with Fig. 11 shows the superiority of the tube wall with circular cross section.

6. RIGID TERMINATION FOR OPEN-CIRCUIT MEASUREMENTS

As has been mentioned above, the rigid termination of measurement specimens for the open-circuit measurement can be achieved by a water column of length $\lambda/4$ for each frequency with a acoustically soft termination behind the specimen, or by a large mass. In the first case, it is inconvenient to have to adjust the length of the water column very accurately for measurements at different frequencies. The rigid termination can be realized with very great accuracy, however. The terminal impedance, which would be infinite for $l = \lambda/4$ without attenuation in the water, is 50 or 20 times as large as the wave impedance when the water column is adjusted for the frequency 10 kc within ±0.43 mm or ±1.2 mm, respectively, which is easily possible.

With the second kind of rigid termination, the use of a large mass, it was necessary to cushion the mass against the measurement tube as shown in Fig. 10, else the tube was very strongly excited into vibration by the sound pressure acting on the surface of the termination, and the termination was acoustically hard only in narrow frequency ranges. For the cushioned attachment, however, the terminal impedance was also smaller than $10\rho c$ in two narrow frequency ranges.

For all measurements that do not require the highest accuracy, a rigid termination in a large frequency range can be achieved for water-borne sound by using a mass.

7. SOUND CONDUCTION BY THE MICROPHONE LEAD STEM

Only microphones that are pure pressure receivers are suitable for the measurement of standing waves in the tube. Especially for the measurement of minima it is essential that the microphone not respond to the particle velocity or the acceleration. Encased Rochelle salt microphones [1] are not suitable for this purpose because they are not acoustically hard enough. On the other hand, microphones consisting of tourmaline crystals of 6 mm diameter [1] have proved very good for this purpose. The investigation of various kinds of microphones showed that the crystals must not be rigidly

connected to the 2-mm-diameter mounting tube that contains the electrical leads. A rigid connection displaces the minima because of the longitudinal vibrations of the tube, and disturbs the level by 40 to 20 dB.

8. EXAMPLES OF MEASUREMENTS OF COMPLEX ACOUSTICAL IMPEDANCE

Measurements of the acoustical impedance in the described tubes have already been published in other places [3][4]. Only a few examples are given here to demonstrate the accuracy of the measurements.

Figure 14 shows the locus of the complex impedance in the frequency range 2 - 20 kc for a resonant sound absorber that has two resonances of the air-filled cavities that it contains. The series resonance circuit whose impedance or admittance is shown in Fig. 15 consists of a metal disk serving as a mass and a rubber disk containing high-resonant-frequency cavities in front of a water column whose length was made equal to $\lambda/4$ at every frequency.

Besides the acoustical impedance of solid bodies with plane boundary surfaces, the impedance of bodies whose dimensions are small compared to the tube radius can also be measured--the diaphragms of underwater sound microphones, for example. For this purpose, the specimens are so arranged in the tube that the center of the diaphragm surface is at the distance $\lambda/4$ from the open end of the tube. The average impedance of the diaphragm surface or surfaces is obtained because the impedance yielded from the sound pressure distribution in the tube is multipled by the ratio of the diaphragm surface to the tube cross section area.

Figure 16 shows the magnitude and phase angle of the impedance of a condenser microphone. The diameter of the bronze diaphragm was 12.5 mm, its thickness 0.4 mm. The microphone is unsuitable for accurate sound pressure measurements near the resonance frequency because it is acoustically soft then. The impedance of encased Rochelle salt microphones was determined in the same manner. The compliant impedance of a good microphone, in which the diaphragm is tightly pressed against the crystal, was so large for 0.2 mm diaphragm thickness up to 30 or 40 kc and for 0.7 mm thickness up to more than 100 kc that the microphone could be considered acoustically hard. All microphones with thin diaphragms, however, have resonances at frequencies above 10 kc that cause a large frequency dependence of the impedance and the sensitivity.

Rochelle salt microphones with small dimensions, despite their poor characteristics, were used where their higher sensitivity (up to 40 times that of tourmaline) was an advantage and a flat frequency response and high acoustical stiffness were not necessary--for example, measurement of transmission attenuation (also in the tube), the recording of directional characteristics, and measurement of reflections in shallow [1] and deep [22] underwater sound measuring tanks.

9. CALIBRATION AND RECORDING OF THE FREQUENCY RESPONSE CURVE OF MICROPHONES IN STANDING OR PROGRESSIVE WAVES

Since sound waves in an appropriately dimensioned measurement tube are completely plane, the sound pressure is constant over the cross section, and the speed is very nearly equal to that of the free medium, a tube is also suitable for comparison measurements on microphones whose cross sections are small compared to the tube diameter, and for the determination of frequency response curves. The measurements can be made with standing waves, or, if the tube is terminated with an absorbing wedge or a resonance absorber, they can be made with progressive waves. As the comparison microphone were used tourmaline pressure receivers whose absolute sensitivity could be computed from the dimensions, the capacitance, and the known piezo modulus, or could be measured in some other manner. So long as the dimensions are smaller than about $\lambda/4$, the sensitivity can be taken as independent of frequency, which means up to 40 kc for 6 mm diameter and length.

10. SPEED OF SOUND MEASUREMENTS

The most accurate sound speed measurements in liquids can be made with ultrasound by optical methods. Accuracy up to 5×10^{-5} can be obtained. Of course this accuracy is attainable only by means of very careful experimental technique in which local temperature increases caused by the quartz source driven at relatively high input power are avoided, and the sound pressure nulls are extrapolated [23]. Ultrasonic measurements are made in small vessels. Measurements can be made in a free sound field by determining the transit time over a large distance. For this, of course, very favorable propagation conditions must exist. For measurements in a tube having a large wall thickness, an accurately defined sound field exists, namely, a perfectly plane wave and very slight attenuation. It is possible to make relative measurements in standing waves by measuring several minimal distances for a measured distance of two wavelengths with an accuracy of $\pm 8 \times 10^{-4}$, if the difference between p_{max} and p_{min} at the measurement frequency amounts to at least 45 dB. For absolute measurements, the accuracy with which the sound speed correction is known enters. It was found that the tolerance obtained by this method is smaller than the discrepancies between the until now most accurately known values of the speed of sound. If, on the other hand, the sound speed is determined by measuring the length of a column of liquid open at one end and a resonance frequency, as has often been done, such an accuracy is difficult to attain because the resonance frequency depends on the terminal impedance of the column, which must be eliminated [9].

Measurements were made in tube No. 1 for a standard frequency of 10 kc (frequency error $< 10^{-8}$) to determine the effect of dissolved air on the sound speed. The wavelengths were measured with an accuracy of $\pm 3.5 \times 10^{-4}$. The difference between p_{max} and p_{min} was 45 - 48 dB (see Fig. 14), so that the measurement accuracy for the sound speed amounted to about $\pm 1 \times 10^{-3}$.

In Fig. 17, curves of the speed of sound in fresh water are plotted from the tables published by the British Admiralty [24] (see also the nomogram taken from these tables by Kalle [25]). In addition, values are plotted from ultrasonic measurements by Giacomini [26], Herbeck [27], Heusinger [28], and Schreuer [23], and several values out of the handbook by d'Ans and Lax [29] that were probably also obtained by ultrasonic measurements. The upper curve was drawn through these measured points. It lies, throughout the whole temperature range, about 5 m/sec above the speed from the tables which were calculated from old values of the thermal compressibility, density, and ratio of the specific heats. A few years after this investigation, Weissler and Del Grosso [30] established the fact that the speed of sound in sea water, which they measured with ultrasound, is about 3-4 m/sec above the values calculated by Kuwahara [31] which agree well with those from [24] for high salinity. The tables are therefore in need of a correction.

The values obtained from the individual sound speed measurements in the tube in the temperature range from 15.5 to 20.5°C were corrected according to Gronwall (Table II) and also shown in Fig. 17. Values that are about 4×10^{-3} smaller would be obtained from the improved formula by Korteweg (Eq. (5)). The sound speed was measured in distilled water and in tap water, and also in air-saturated water (highest measured air content 2.1%) and in water that had been deaerated by long boiling or evacuation (smallest air content 0.4%). The measured points fall on the average around 1.0 to 2.2 m/sec, thus about 0.7 to 1.5×10^{-3} below the curve of the ultrasonic measurements. The deviations of the individual measured points from the dotted curve amount to at most 10^{-3}. Within these limits of measurement error, no effect of the air content was found. Herbeck [27] has measured with a Pierce interferometer an increase of the sound speed of saturated compared to unsaturated water of about 10^{-4} at 815 kc.

These measurements show that rather accurate sound speed measurements can be made in liquids in thick-walled tubes at audio frequencies after a slight increase of the correction.

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Table I

p_{st}/p_e	Smallest possible pressure ratio p_{max}/p_{min}		Largest possible displacement σ_{st}
--	--	dB	--
0.005	400	52	0.0015
0.01	200	46	0.003
0.02	100	40	0.006
0.05	40	32	0.016

Table II

Tube No.	Material	r_1 (mm)	r_2 (mm)	h/r_1	Inner cross section (mm^2)	l (mm)	c'/c_0 meas.	c'/c_0 Eq. (5)	c'/c_0 Gronwall	Transverse resonance (kc)
1	steel	25.0	40.0	0.600		400	0.971	0.971	0.972	17.4
2	steel		40.0		32.5x32.5	500	0.972	0.980	0.976	22.8
3	brass	19.0	25.0	0.315		350	0.907	0.908	0.905	22.8

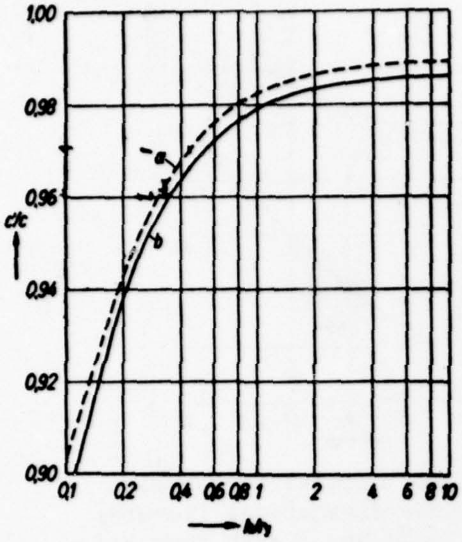

Fig. 1. Sound speed in a water-filled steel tube as a function of the ratio of the wall thickness to the inner radius. Curve a: calculated from Eq. (5); curve b: calculated according to Gronwall.

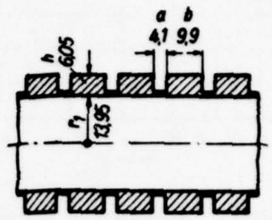

Fig. 2. Dimensions of a tube with subdivided wall.

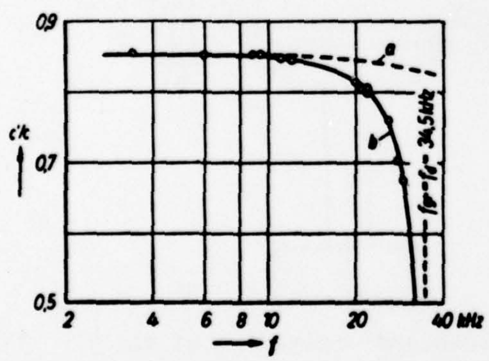

Fig. 3. Sound speed in the tube according to Fig. 2 as a function of the frequency. Curve a: calculated (wall as a compliance); curve b: calculated (wall with compliance and mass). o o *measured points.*

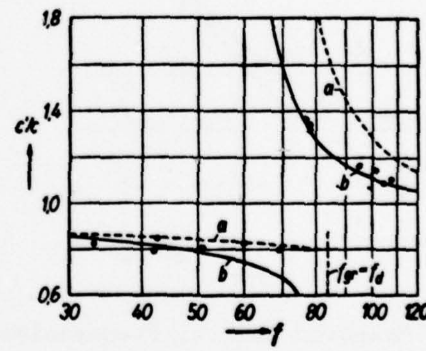

Fig. 4. Sound speed of the plane wave and the first higher order of waves in a petroleum-filled glass tube as a function of the frequency. Curves a: calculated (wall as a compliance); curves b: calculated (wall with compliance and mass); o o measured points from Field and Boyle.

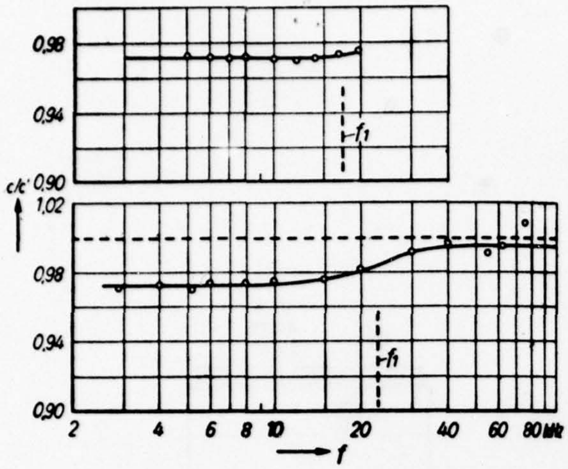

Fig. 5 (Top). Measured sound speed in tube No. 1 as a function of the frequency; f_1 = frequency of the lowest radial resonance.

Fig. 6 (Bottom). Measured sound speed in tube No. 2 as a function of the frequency; f_1 = frequency of the lowest radial resonance.

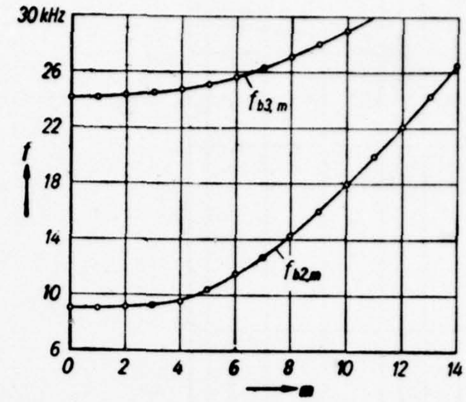

Fig. 7. Measured natural frequencies of two- and three-dimensional flexural vibrations of tube No. 3 with four and six nodes on the circumference as a function of the number m of the nodes on the tube length.

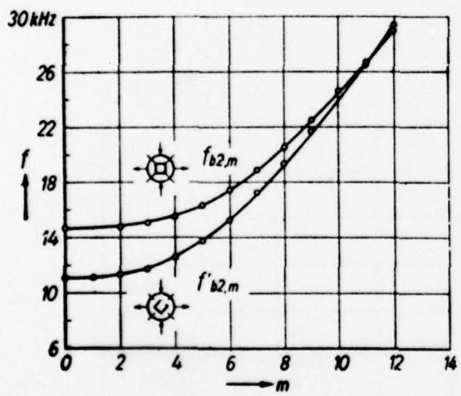

Fig. 8. Measured natural frequencies of the two types of two- and three-dimensional flexural waves of tube No. 2 with four nodes on the circumference as a function of the number of nodes on the tube length.

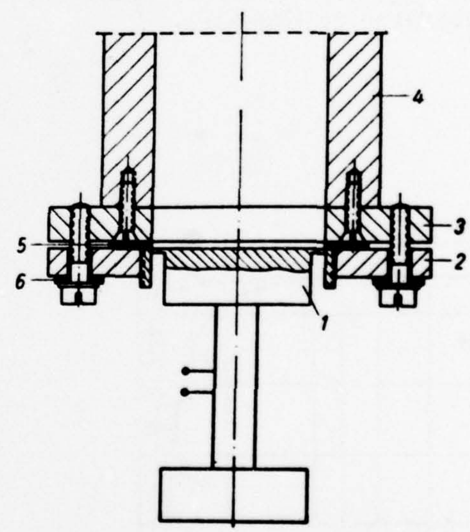

Fig. 9. Cushioned fastening of the source to the tube. 1 Magnetostriction source; 2,3 brass rings; 4 tube; 5,6 rubber rings.

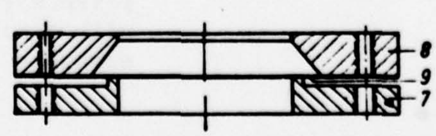

Fig. 10. Supplementary filter for structure-borne sound. 7,8 bronze rings; 9 brazed bronze sheet.

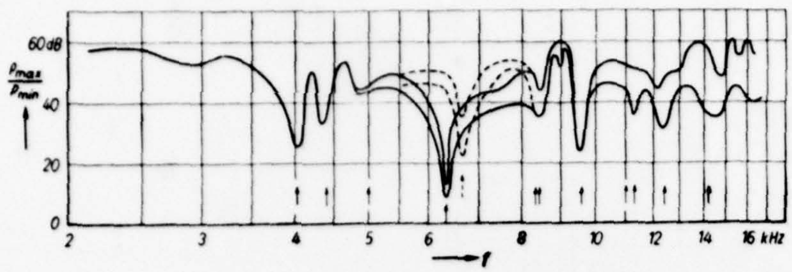

Fig. 11. Measured level difference between the maxima and minima in the open-ended tube No. 1. Upper curves: best values; lower curves: poorest values. Solid lines without supplementary structure-borne-sound filter; dotted lines with filter. Arrows, measured natural frequencies of the tube wall.

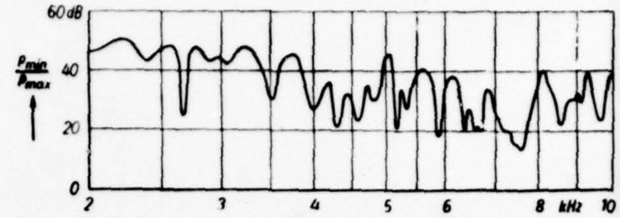

Fig. 12. Measured level difference between the maxima and the minima in tube No. 3.

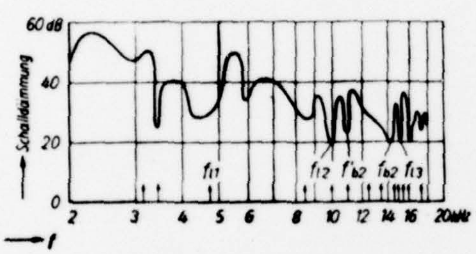

Fig. 13. Sound insulation between the upper and the lower parts of tube No. 2, coupled only through the tube wall, and without the filter shown in Fig. 10. Arrows: measured natural frequencies of the tube wall; f_{11}, f_{12}, f_{13}, lowest orders of longitudinal vibrations; f_{b2} and f_{b2}', lowest orders of the three-dimensional flexural vibrations.

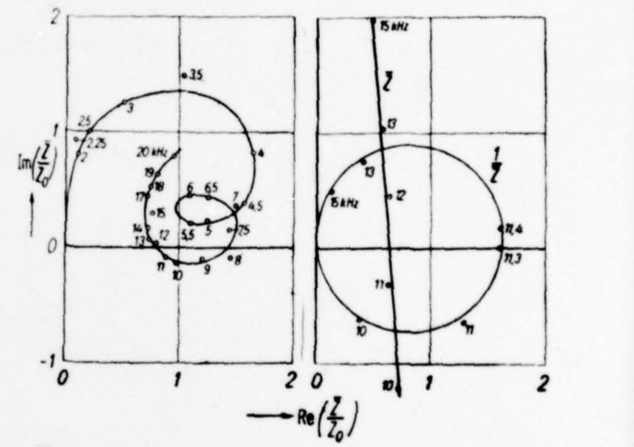

Fig. 14. Measured complex impedance of a resonance sound absorber for air termination (short circuit) in the frequency range 2 to 20 kc.

Fig. 15. Measured complex impedance or admittance of an acoustical series resonant circuit (15-mm-thick iron disk in front of 4-mm-thick rubber disk containing small air-filled cavities, and rigid termination).

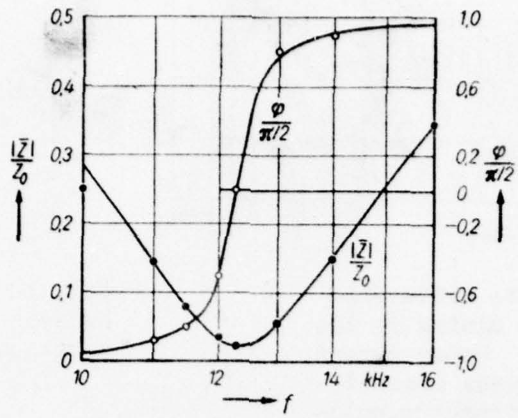

Fig. 16. Magnitude and phase angle of the acoustical impedance of a condenser microphone as a function of the frequency.

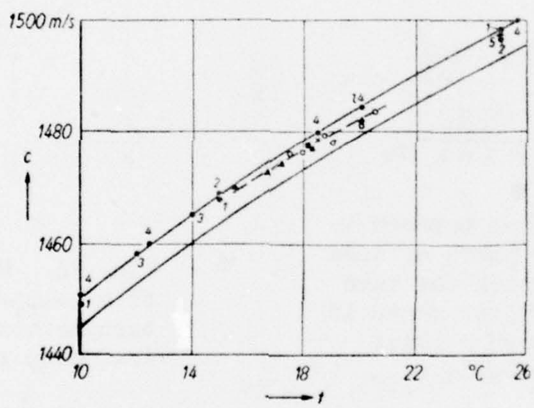

Fig. 17. Speed of sound in free water as a function of temperature according to the tables of the British Admiralty (lower curve), ultrasonic measurements (upper curve, 1 d'Ans and Lax, 2 Giacomini, 3 Herbeck, 4 Heusinger, 5 Schreuer), and measurements at 10 kc in a tube for various air contents (middle curve, ▲ distilled water, 2% air; ■ distilled water, 0.5% air; o tap water, 2.0% air; x tap water, 0.5% air); Gronwall correction applied.